WHO LIKES HOMEWORK

Question of the Day

Problem: Integrate $\int \frac{x^2}{(x+1)^3}dx$

Solution:

Turn the integrand, $\frac{x^2}{(x+1)^3}$, into a sum of fractions. Thus, this becomes

$$\frac{x^2}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} \Rightarrow x^2 = A(x+1)^2 + B(x+1) + C$$

If $x = -1$, this becomes => $(-1)^2 = 0A + 0B + C$ => C = 1

x may take on any value, so arbitrarily pick two values for x:

Let $x = 1$, then we have $(1)^2 = 2^2 A + 2B + 1 => 1 = 4A + 2B + 1$ eq 1
Let $x = 2$, then we have $(2)^2 = 3^2 A + 3B + 1 => 4 = 9A + 3B + 1$ eq 2

Take these two equations to solve A and B simultaneously to find $A = 1$ and $B = -2$

So, after plugging in the values for A, B, and C we now have

$$\int \frac{x^2}{(x+1)^3}dx = \int \frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{1}{(x+1)^3}dx = \int \frac{1}{x+1}dx - \int \frac{2}{(x+1)^2}dx + \int \frac{1}{(x+1)^3}dx$$

Integrating these terms gives

$$\ln|x+1| + \frac{2}{(x+1)} - \frac{1}{2(x+1)^2} + C$$

Therefore, $\int \frac{x^2}{(x+1)^3}dx = \ln|x+1| + \frac{2}{(x+1)} - \frac{1}{2(x+1)^2} + C$