WHO LIKES HOMEWORK

QUESTION OF THE DAY

Problem: A solid cylinder of radius R sits atop a ramp of angle θ, a height H above the ground and a ramp length D. Calculate the velocity, v, the cylinder will be moving when it reaches the bottom of the ramp.

Solution

The idea here is to use the conservation of energy.

$KE = PE$ The kinetic energy is going to equal the *total energy involved with movement*, which is linear kinetic energy and the rotational kinetic energy.

$KE_L = \frac{1}{2}mv^2$ and $KE_R = \frac{1}{2}I\omega^2$ and PE=mgH

$\omega =$ Angular Velocity
$g =$ Force of Gravity
$I =$ Moment of Inertia

Therefore, we have

$$\frac{1}{2}mv^2 + \frac{1}{2}I\varpi^2 = mgH$$

For a solid cylinder rotating around its axis, the moment of inertia equal to $I = \frac{1}{2}mR^2$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\varpi^2 = mgH$$

$$\frac{1}{2}v^2 + \frac{1}{4}R^2\omega^2 = gH$$

Finally, we will use the definition of angular velocity, $\omega = \frac{v}{R}$

$$\frac{1}{2}v^2 + \frac{1}{4}R^2\left(\frac{v}{R}\right)^2 = gH$$

$$\frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2 = gH$$

Solving for v gives $v = \sqrt{\frac{4gH}{3}}$