

QUESTION OF THE DAY

Problem: Multiply out $(x-4)^4$

Solution: There are two ways to do this

Way 1

$$\begin{aligned}
 (x-4)^4 &= (x-4)^2(x-4)^2 \\
 &= (x^2 - 8x + 16)(x^2 - 8x + 16) \\
 &= x^2(x^2 - 8x + 16) - 8x(x^2 - 8x + 16) + 16(x^2 - 8x + 16) \\
 &= x^4 - 8x^3 + 16x^2 - 8x^3 + 64x^2 - 128x + 16x^2 - 128x + 256 \\
 &= x^4 - 16x^3 + 96x^2 - 256x + 256
 \end{aligned}$$

Way 2

We will use a method called Pascal's Triangle. It is a modified version of the Binomial Theorem. It applies to any binomial of the form

$$(x + y)^n = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \dots + a_ny^n$$

To determine all of the coefficients a , go from left to right and start with a_0 . So, $a_0=1$ and $a_1=4$ and so on.

Pascal's Triangle

N=0					1					
N=1				1	1					
N=2			1	2	1					
N=3	1			3	3	1				
N=4	1	4			6	4	1			

For our situation, we have $(x-4)^4$, where $x=x$ and $y=-4$. Plugging these into the definition, and using Pascal's Triangle, we have

$$\begin{aligned}
 (x-4)^4 &= 1 \cdot x^4 + 4(x^3)(-4) + 6(x^2)(-4)^2 + 4(x)(-4)^3 + 1(-4)^4 \\
 (x-4)^4 &= x^4 - 16x^3 + 96x^2 - 256x + 256
 \end{aligned}$$

Notice that both ways yield the same answer

$$(x-4)^4 = x^4 - 16x^3 + 96x^2 - 256x + 256$$